GUIDED WAVE PROPAGATION IN UNCERTAIN ELASTIC MEDIA THROUGH STOCHASTIC DIFFUSION MATRIX

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ABSTRACT

In this paper, the authors present a numerical approach to study Defect detection through Stochastic Wave Finite Element Method. The uncertain material properties are modeled as a set of random fields. The structure is presented considering two waveguides connected through a stochastic coupling element, simulated as the defect (crack). Diffusion matrix for uncertain media through stochastic wave finite element method is studied in this paper. The forced response following a vibratory excitation is computed to investigate the defect detection. The computational efficiency of the method is demonstrated by comparison with MC simulation.
1 INTRODUCTION

Many researchers proposed some structural health monitoring (SHM) techniques in order to carry out the monitoring and the diagnosis of the risks [1, 2]. SHM is among the fields of application of guided wave propagation. Wave finite element method (WFEM) can be used for wave propagation predictions and wave scattering estimations. The WFEM regards the waveguide structure as a periodic system assembled by identical substructures, the dispersion curves and the mode shapes are among the primary properties to be given [3, 4]. In the literature, however, most of founded numerical issues of wave propagation simulations are mainly limited to deterministic media. Numerical guided wave techniques characterization in spatially homogeneous random media is investigated in this paper. The uncertainties are often present in geometric properties, material characteristics and boundary conditions of the model. These variables are taken into account in models according to the both parametric [5] and non-parametric [6] approaches. Ichchou et al [7] considered the wave propagation features in random guided elastic media through the Stochastic Wave Finite Element Method (S.W.F.E.M) using a parametric probabilistic technique. Bouchoucha et al [8] presented in their paper a numerical approach to study the guided elastic wave propagation in uncertain elastic media. Stochastic Wave Finite Element Method (S.W.F.E.M) formulation with consideration of spatial variability of material and geometrical properties is developed for probabilistic analysis of structures. They extended their work in [9] to the diffusion matrix for uncertain media through stochastic wave finite element method (SWFEM). The stochastic diffusion relationship allows evaluating the statistics of reflection and transmission coefficients under structural uncertainty.

This paper will extends mentioned works in order to provide a full numerical description of the stochastic problem of the wave propagation in uncertain damaged structure. The main contribution of this paper seems the calculation of the stochastic forced response of the uncertain waveguide with defect.

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The damaged structure is modelled by 2 periodic waveguides connected through a coupling element assimilated as a defect (figure 1). The stochastic parameters have a Gaussian distribution through the first order perturbation.

![Figure 1. An illustration of the damaged structure](image)

We consider the stochastic state vectors: \( \bar{u}_L = (\bar{q}_L^T - \bar{f}_L^T)^T \) and \( \bar{u}_R = (\bar{q}_R^T - \bar{f}_R^T)^T \), we have:

\[
\bar{u}_L = \tilde{S}_L \cdot \bar{u}_L
\]

Using a modal decomposition [8], the state vectors can be projected on the wave mode base as:

\[
\bar{u}_{L,(i)} = \phi \; Q_{L,(i)}^{(i)} \quad \text{and} \quad \bar{u}_{R,(i)} = \phi \; Q_{R,(i)}^{(i)}
\]

\[
\begin{pmatrix}
\tilde{Q}_{L,N}^{ref} \\
\tilde{Q}_{R,N}^{ref}
\end{pmatrix} = \tilde{X}^{-1} \begin{pmatrix}
\tilde{Q}_{L,N}^{ref} \\
\tilde{Q}_{R,N}^{ref}
\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}
\tilde{Q}_{L,N}^{ref}^{(i)} \\
\tilde{Q}_{N,2L}^{ref}^{(i)}
\end{pmatrix} = \tilde{X}^{-1} \begin{pmatrix}
\tilde{Q}_{L,N}^{ref}^{(i)} \\
\tilde{Q}_{N,2L}^{ref}^{(i)}
\end{pmatrix}
\]

Where \( \tilde{Q}^{(i)} \) is the vector of the stochastic wave mode amplitude, \( \tilde{\phi} \) is the matrix of the stochastic
eigenvectors, $\vec{\lambda}$ is a diagonal stochastic eigenvalue matrix. Using the wave/defect interaction, the diffusion matrix is written as [9]:

$$
\begin{pmatrix}
\tilde{Q}_N^r \\
\tilde{Q}_{LN}^r
\end{pmatrix} = \begin{pmatrix}
\tilde{D}^c_{LL} \phi_q^{inc} + \phi_q^{ref} & \tilde{D}^c_{LR} \phi_q^{inc} + \phi_q^{ref} \\
\tilde{D}^c_{RL} \phi_q^{inc} + \phi_q^{ref} & \tilde{D}^c_{RR} \phi_q^{inc} + \phi_q^{ref}
\end{pmatrix}\begin{pmatrix}
\tilde{Q}_N^{inc} \\
\tilde{Q}_N^{ref}
\end{pmatrix}
$$

(4)

Where $\tilde{D}^c$ is the stochastic dynamic stiffness matrix of the coupling element. The stochastic displacement $\tilde{q}_L^{(k)}$ can be calculated as follows:

$$
\tilde{q}_L^{(k)} = \tilde{\phi}_q^{inc} (\tilde{\lambda}^{inc})^{(k-1)} \tilde{Q}_N^{inc(1)} + \tilde{\phi}_q^{ref} (\tilde{\lambda}^{ref})^{(k-1)} \tilde{Q}_N^{ref(1)}
$$

(5)

Where $\tilde{Q}_N^{(1)}$ is already calculated by adopting the appropriate boundary conditions.

### 3 NUMERICAL RESULTS AND DISCUSSION

In this section, we study the longitudinal vibration of the structure in order to validate the SWFEM. We consider the following boundary conditions:

$$
\tilde{F}_L^{(1)} = \tilde{F} \text{ (excitation) and } \tilde{F}_R^{(N)} = 0 \text{ (free end)}
$$

(6)

The excitation has a Gaussian distribution $\tilde{F} = \tilde{F} + \sigma_F \cdot \epsilon$ when $\sigma_F = 0.05. \tilde{F}$.

We introduce the uncertainty in the structural parameters to study their effects on the wave propagation. These parameters were a Gaussian distribution (standard deviation = 0.05.mean).

We study the forced response of the structure with defect for the longitudinal mode. Monte Carlo simulations are used to validate the SWFEM results. The mean of the stochastic forced response with defect is presented in figure 2. The effect of the structural parameter perturbations in presence of defect is presented in figure 3. In figure 4, we present the mean and the standard deviation of the forced response with defect following uncertainty introduced in Young modulus in order to demonstrate the efficiency of the proposed method. In fact, the wave propagation isn’t affected by the perturbation of the structural parameters. Figure 5 illustrates the competence of the SWFEM as a tool for the defect detection through the comparison between the forced response with and without defect.

![Figure 2. Mean of the forced response of the waveguide with defect](image1.png)

![Figure 3. Standard deviation with defect (E stochastic)](image2.png)

![Figure 4. The forced response with defect (E stochastic)](image3.png)

![Figure 5. The defect detection for the traction compression mode](image4.png)
4 CONCLUSION

In this paper, the subject of forced response in uncertain structure with defect was dealt with. The structure is presented considering two waveguides connected through a stochastic coupling element, simulated as the defect (crack), the stochastic diffusion relationship is used to study the wave/defect interaction following an uncertainty introduced in the structure parameters. The stochastic wave finite element method is used for the defect detection in uncertain structure. The main paper finding can be extracted as follows:
1. Stochastic forced response associated to stochastic wave propagation is given.
2. Stochastic diffusion matrix associated to damage front is given. This paper provides a numerical investigation of guided waves and defect interaction.
3. The defect detection in uncertain structure is offered.

The SWFE offers some interesting research perspectives. The use of SWFE for energy issues in complex wave guide is an important task. Among the investigations in progress, the mid and high frequencies behavior is the main target in this case. Further investigations are under progress in order to use such numerical methods in the context of smart materials and structures.

REFERENCES