THE WAVE BASED METHOD: CURRENT STATE OF THE ART

E. Decker*1, O. Atak1, L. Coxx1, R. D’Amico1, H. Devriendt1, S. Jonckheere1, K. Koo1, B. Pluymers1, D. Vandeputte1 and W. Desmet1

1KU Leuven, Department of Mechanical Engineering
Celestijnenlaan 300B box2420, Heverlee, BELGIUM
Email: elke.deckers@mech.kuleuven.be, wim.desmet@mech.kuleuven.be

ABSTRACT

Most commonly, element based prediction techniques, such as the Finite Element Method and the Boundary Element Method are used to solve steady state dynamic problems. These procedures divide the problem domain or its boundary into a large number of small elements in which the dynamic fields are described using simple, most often polynomial, shape functions. When frequency increases and the wavelengths shorten, the mesh needs to be refined to obtain accurate solutions, due to interpolation and pollution errors. The sizes of the system matrices, and consequently the computational load, increase accordingly. From a certain frequency on, the models become computationally too demanding, such that the element based techniques are limited for solution at lower frequencies.

The Wave Based Method is also a deterministic numerical prediction technique to solve steady-state dynamic problems, developed to overcome some of these frequency limitations imposed by the use of small elements and simple interpolation functions. The method belongs to the family of indirect Treffitz approaches and uses a weighted sum of so-called wave functions, which are exact solutions of the governing partial differential equations, to approximate the dynamic field variables. Contrarily to element based techniques, the problem domain is subdivided into a small number of large, convex subdomains. By minimising the errors on boundary and interface conditions, a system of equations is obtained which can be solved for the unknown contribution factors of each wave function. As a result, the system of equations is smaller and a higher convergence rate and lower computational loads are obtained as compared to conventional prediction techniques. On the other hand, the method shows its full efficiency for moderately complex geometries. Various enhancements have been made to the method through the years, in order to extend the applicability of the Wave Based Method. This paper aims to give an overview of the current state of the art of the Wave Based Method, its modelling procedure, application areas and extensions to the method such as hybrid and multi-level approaches.
1 INTRODUCTION

The Wave Based Method (WBM) [1] belongs to the family of Trefftz approaches. It is designed for solving steady-state dynamic problems, governed by a (set of) Helmholtz equation(s) and can be applied to bounded as well as (semi-)unbounded problem domains. This paper gives an overview of the current state of the art of the WBM, summarising fifteen years of research. The first section describes which problem types can be solved with the WBM. Next, the four steps involved in the WB modelling procedure are discussed. Finally, the application areas of the WBM and hybrid and multi-level extensions are discussed.

2 GENERALISED HELMHOLTZ PROBLEM

Consider a general interior/exterior steady-state dynamic problem. It is assumed that the mathematical formulation of the physics inside the problem domain $\Omega$ gives rise to, or can be cast into $N_H$ (modified) Helmholtz equations. The problem boundary $\Gamma = \partial \Omega$ consists of two parts in case the problem domain is unbounded: the finite part of the boundary, $\Gamma_b$, and the boundary at infinity, $\Gamma_\infty$. The finite part of the boundary can be divided into non-overlapping parts: $\Gamma_b = \bigcup_i \Gamma_i$, on which different boundary conditions can be imposed. On each point of the boundary, $N_H$ boundary conditions need to be defined to obtain a well-posed problem. At the boundary at infinity $\Gamma_\infty$, non-reflecting boundary conditions are imposed.

3 THE WBM MODELLING PROCEDURE

The general modelling procedure of the WBM [1] to solve a generalised Helmholtz problem consists of four steps, which are detailed next.

3.1 Partitioning of the problem domain

When applied to bounded problems, the convexity of the considered domain is a sufficient condition for the WB approximations to converge towards the exact solution of the problem under study [1]. When the considered problem domain is non-convex, it is, in a first step, partitioned into a number of convex subdomains. When applied to unbounded problems, an initial partitioning of the unbounded domain into a bounded and an unbounded region by a truncation curve $\Gamma_t$ precedes the partitioning into convex subdomains [2]. $N_H$ continuity conditions are imposed for each subdomain on the interfaces between two adjacent subdomains.

3.2 Field variable expansion

The $N_H$ steady-state dynamic field(s) are approximated in each subdomain by a solution expansion of wave functions which are exact solutions of the associated Helmholtz equation. In case a non-homogeneous Helmholtz equation is considered, a particular solution is added to the wave function set. In case of an unbounded subdomain, the wave functions are selected to additionally inherently fulfill the non-reflecting boundary condition at $\Gamma_\infty$. The applied sets of wave functions within the WBM can be found in the theoretical sections of the papers cited in Section 4.

3.3 Construction of the system of equations

Within each subdomain, the proposed solution expansion always exactly satisfies the Helmholtz equation(s), irrespective of the values of the unknown contribution factors. The matrix system of
equations is constructed by minimising the errors on the boundaries and interfaces, by applying a weighted residual approach.

3.4 Solution and postprocessing

In a final step, the system matrix of equation is solved for the unknown wave function contribution factors. The backsubstitution of these values in the field variable expansions leads to an analytical expression of the approximation of the field variables in each of the subdomains. Derivative quantities can be easily obtained by applying differential operators to the wave function sets. Since also evanescent components are used in the wave function sets, also near field effects can be captured.

4 WBM STATE OF THE ART

The concept of the WBM was introduced by Desmet [1] and since then it has been the topic of continuous research. The method has been applied to interior [3] and exterior [4] acoustic problems, plate bending and plate membrane problems [5], assemblies of flat shells [6], coupled vibro-acoustic problems [7] and for poroelastic materials [8]. For each of the problems a comparison has been made to standard element-based prediction techniques. The WBM shows a better efficiency for problems of a low to moderate geometrical complexity

4.1 Multi-level WBM

In the case an unbounded problem geometry contains several scatterers, the WBM loses its attractiveness. The truncation $\Gamma_i$ needs to enclose all scatterers and inside this truncation, the convexity requirement may lead to a very large number of subdomains. When, for instance, a number of circular scatterers are considered, it is even impossible to obtain convex subdomains. The same holds for a bounded subdomain with (a number of) inclusion(s). To overcome these difficulties, the concept of multi-level modelling was introduced considering each scatterer or inclusion in a separate level [9].

4.2 Hybrid WB methods

The second class of enhancements applies the combined use of two numerical methods to one problem under study. Each method is applied according to its own strengths. This way, a strong hybrid approach can be obtained, taking benefit of the best properties of two approaches. In these lines of thought, FE-WBM [10], BE-WBM and WBM-SEA [11] have been developed.

5 CONCLUSION

This paper discusses the state of the art of the WBM, summarising fifteen years of continuous research. The WB modelling procedure is presented and current application areas and extensions of the method are explained. The presentation will show the performance of the method with respect to element based prediction techniques for a number of applications.

ACKNOWLEDGMENTS

The institute for Promotion of Innovation by Science and Technology Flanders (Belgium) (IWT-Vlaanderen) is gratefully acknowledged for the support of the doctoral research of Laurens Coox, Hendrik Devriendt and Stijn Jonckheere. The authors also gratefully acknowledge
the European Commission for their support through the ITN Marie Curie project GA-290050 GRESIMO and the EID Marie Curie project GA-316422 eLiQuiD. The IWT Flanders within the ASTRA and the HEV-NVH project, the Fund for Scientific Research - Flanders (F.W.O.), Belgium and the Research Fund KU Leuven are also gratefully acknowledged.

REFERENCES


